Real-time Scheduling of Aperiodic and Sporadic Tasks (2)
Lecture outline

• Scheduling aperiodic jobs (cont’d)
  • Simple sporadic server

• Scheduling sporadic jobs
Limitations of Deferrable Servers

- Limitation of deferrable servers – they may delay lower-priority tasks for more time than a periodic task with the same period and execution time:

![Diagram showing budget replenishment and task blocking](image-url)

- $T_{DS} = (p=3, e=1)$
- $T_1 = (\phi=2, p=3.5, e=1.5)$
- $T_2 = (p=6.5, e=0.5)$

$T_1$ blocked for 1.2 units although execution time of deferrable server is only 1.0 units.
Sporadic Servers

• A sporadic server is designed to eliminate this limitation
  • A different type of periodic server: several different sub-types
  • More complex consumption and replenishment rules ensure that a sporadic server with period $p_s$ and budget $e_s$ never demands more processor time than a periodic task with the same parameters
Simple, Fixed-priority, Sporadic Server (1)

- System, $T$, of independent preemptable periodic tasks and a sporadic server with parameters $(p_s, e_s)$
  - Fixed-priority scheduling; system can be scheduled if sporadic server behaves as a periodic task with parameters $(p_s, e_s)$

- Define:
  - $T_H$: the periodic tasks with higher priority than the server (may be empty)
  - $t_r$: the last time the server budget replenished
  - $t_f$: the first instant after $t_r$ at which the server begins to execute
  - At any time $t$ define:
    - $BEGIN$ as the start of the earliest busy interval in the most recent contiguous sequence of busy intervals of $T_H$ starting before $t$ (busy intervals are contiguous if the later one starts immediately the earlier one ends)
    - $END$ as the end of the latest busy interval in this sequence if this interval ends before $t$; define $END = \infty$ if the interval ends after $t$
Simple, Fixed-priority, Sporadic Server (2)

- **Consumption rule:**
  - At any time $t \geq t_r$, if the server has budget and if either of the following two conditions is true, the budget is consumed at the rate of 1 per unit time:
    - C1: The server is executing
    - C2: The server has executed since $t_r$ and $END < t$
  - When they are not true, the server holds its budget

- **That is:**
  - The server executes for no more time than it has execution budget
  - The server retains its budget if:
    - A higher-priority job is executing, or
    - It has not executed since $t_r$
  - Otherwise, the budget decreases when the server executes, or if it idles while it has budget
Simple, Fixed-priority, Sporadic Server (2)

- **Replenishment rules**

  - **R1:** When system begins executing, and each time budget is replenished, set the budget to $e_S$ and $t_r = \text{the current time}$.

  - **R2:** When server begins to execute (defined as time $t_f$)
    
    \[
    \begin{align*}
    \text{if } END = t_f & \text{ then } \\
    t_e &= \max(tr, BEGIN) \\
    \text{else if } END < t_f & \text{ then } \\
    t_e &= t_f
    \end{align*}
    \]

    The next replenishment time is set to $t_e + p_S$

  - **R3:** budget replenished at the next replenishment time, unless:
    
    \[
    \begin{itemize}
    \item If $t_e + p_S$ is earlier than $t_f$ the budget is replenished as soon as it is exhausted
    \item If $T$ becomes idle before $t_e + p_S$, and becomes busy again at $t_b$, the budget is replenished at $\min(t_b, t_e + p_S)$
    \end{itemize}
    \]

\[t_e = \text{effective replenishment time}\]
Example

$T_1 = (3, 0.5), \ T_2 = (4, 1.0), \ T_3 = (19, 4.5), \ T_{ss} = (5, 1.5)$

Rate monotonic schedule; simple sporadic server

Max. blocking time due to sporadic server = 1.5

$A_1$: $r = 3, \ e = 1$

$A_2$: $r = 7, \ e = 2$

$A_3$: $r = 15.5, \ e = 2$
Example

Job $A_1$ executes

Job $A_1$ released, server blocked

No aperiodic jobs, server suspended

No budget

Sporadic server is constrained to execute for at most 1.5 units out of every 5, due to consumption and replenishment rules

Job $A_2$ released but no budget

Budget available but blocked

Job $A_2$ executes

Budget continues to be used according to rule C2
Correctness of Schedule

• More complex than a polling server or a deferrable server, but much easier to prove the system can be scheduled

• Theorem: for the purpose of validating a schedule, you can treat a simple sporadic server \((p_s, e_s)\) in a fixed-priority system exactly the same as any other periodic task \(T_i\) with \(p_i = p_s\) and \(e_i = e_s\)

• The actual inter-release times of the sporadic server will sometimes be greater than \(p_s\), and their execution times less than \(e_s\), but this does not affect correctness
Simple, Dynamic-priority, Sporadic Server

• It is possible to define a simple sporadic server to operate in a dynamic-priority environment
  • E.g., when using EDF or LST scheduling

• Consumption and replenishment rules conceptually similar to those for a fixed-priority scheduler, with minor modifications that account for the difference in scheduling algorithm
  • Provides same scheduling guarantees as the simple sporadic server for fixed-priority schedulers
  • A simple sporadic server \((p_s, e_s)\) in an EDF or LST system can be treated exactly the same as any other task \(T_i\) with \(p_i = p_s\) and \(e_i = e_s\) when testing whether the system can be scheduled
Scheduling Sporadic Jobs

• Consider the problem of scheduling sporadic jobs alongside a system of periodic tasks and aperiodic jobs

• Recall the sporadic job scheduling problem:
  • Based on the execution time and deadline of each newly arrived sporadic job, decide whether to accept or reject the job
  • Accepting the job implies that the job will complete within its deadline, without causing any periodic task or previously accepted sporadic job to miss its deadline
  • Do not accept a sporadic job if cannot guarantee it will meet its deadline
Model for Scheduling Sporadic Jobs

- When sporadic jobs arrive, they are both accepted and scheduled in EDF order
  - In a dynamic-priority system, this is the natural order of execution
  - In a fixed-priority system, the sporadic jobs are executed by a periodic server that performs an acceptance test, and runs the sporadic jobs in EDF order
  - In both cases, no new scheduling algorithm is required

- Definitions:
  - Sporadic jobs are denoted by $S_i(r_i, d_i, e_i)$ where $r_i$ is the release time, $d_i$ is the (absolute) deadline, and $e_i$ is the maximum execution time
  - The density of a sporadic job $\Delta_i = e_i / (d_i - r_i)$
    - The total density of a system of $n$ jobs is $\Delta = \Delta_1 + \Delta_2 + \ldots + \Delta_n$
  - The job is active during its feasible interval $(r_i, d_i]$
Sporadic Jobs in Dynamic-Priority Systems

• Theorem: A system of independent preemptable sporadic jobs can be scheduled using EDF if the total density of all active jobs in the system $\leq 1$ at all times
  • This is the standard scheduling test for EDF systems, but including both periodic and sporadic jobs
  • This test uses the density since deadlines may not equal periods; hence it is a sufficient test, but not a necessary test

• What does this mean?
  • If we can bound the frequency with which sporadic jobs appear to the running system, we can guarantee that none are missed
  • Alternatively, when a sporadic job arrives, if we deduce that the total density would exceed 1 in its feasible interval, we reject the sporadic job (admission control)
Admission Control for Sporadic Jobs/EDF

- At time $t$ there are $n$ active sporadic jobs, stored in non-decreasing order of deadline
  - The deadlines partition the time from $t$ to $\infty$ into $n + 1$ discrete intervals: $I_1, I_2, \ldots, I_{n+1}$
    - $I_1$ begins at $t$ and ends at the earliest sporadic job deadline
    - For each $1 \leq k \leq n$, each interval $I_{k+1}$ begins when the interval $I_k$ ends, and ends at the next deadline in the list (or $\infty$ for $I_{n+1}$)
  - The scheduler maintains the total density $\Delta_{s,k}$ of each interval $I_k$

- Let $I_l$ be the interval containing the deadline $d$ of the new sporadic job $S(t, d, e)$
  - The scheduler accepts the job if $\frac{e}{d-t} + \Delta_{s,k} \leq 1 - \Delta$
    - i.e. accept if the new sporadic job can be added, without increasing the density of any intervals past 1
Admission Control for Sporadic Jobs/EDF

- Notes:

  - This acceptance test is *not* optimal: a sporadic job may be rejected even though it could be scheduled (the result for the maximum utilisation is based on the density and hence is sufficient but not necessary).
  
  - It is possible to derive a – much more complex – expression taking into account slack time, that is optimal. Unclear if the complexity is worthwhile.
  
  - This acceptance test assumes every sporadic job is ready for execution when released

    - If this is not the case, must modify the acceptance test to take into account the time when the jobs become ready, rather than their release time, when testing the intervals to see if their density exceeds 1.
Sporadic Jobs in Fixed-Priority Systems

• Use a sporadic server to execute sporadic jobs in a fixed-priority system
  • The server \((p_s, e_s)\) has budget \(e_s\) units every \(p_s\) units of time, so the scheduler can compute the least amount of time available to every sporadic job in the system
  • Assume that sporadic jobs ordered among themselves in EDF
  • When first sporadic job \(S_1(t, d_{s,1}, e_{s,1})\) arrives, there is at least \(\left\lfloor (d_{s,1} - t)/p_s \right\rfloor e_s\) units of processor time available to the server before the deadline of the job
  • \(\left\lfloor (d_{s,1} - t)/p_s \right\rfloor\) = number of server periods available
  • Therefore it accepts \(S_1\) if slack of job \(\sigma_{s,1}(t) = \left\lfloor (d_{s,1} - t)/p_s \right\rfloor e_s - e_{s,1} \geq 0\)

Time available
Execution time

[cont’d]
Sporadic Jobs in Fixed-Priority Systems

• To decide if a new job $S_i(t, d_{s,i}, e_{s,i})$ is acceptable when there are $n$ sporadic jobs in the system, the scheduler first computes the slack $\sigma_{s,i}(t)$ of $S_i$: 

$$
\sigma_{s,i}(t) = \lfloor (d_{s,i} - t)/p_s \rfloor e_s - e_{s,i} - \sum_{d_{s,k} < d_{s,i}} (e_{s,k} - \xi_{s,k})
$$

where $\xi_{s,k}$ is the execution time of the completed part of the existing job $S_k$

The job cannot be accepted if $\sigma_{s,i}(t) < 0$

• As for $\sigma_{s,1}(t)$, but accounting for the already accepted sporadic jobs

• If $\sigma_{s,i}(t) \geq 0$, the scheduler then checks if any existing sporadic job $S_k$ with deadline after $d_{s,i}$ may be adversely affected by the acceptance of $S_i$

• Check if the slack $\sigma_{s,k}(t)$ for each $S_k$ at the time is at least equal to the execution time $e_{s,i}$ of $S_i$ (i.e., $S_i$ is accepted if $\sigma_{s,k}(t) - e_{s,i} \geq 0$ for every existing sporadic job $S_k$ with deadline $\geq d_{s,i}$)

• This acceptance test for fixed-priority systems is more complex than that for dynamic-priority systems, but is still of reasonable time complexity to be implemented “on-line”
Practical Usage

• Hybrid sporadic/background server included in real time extensions to POSIX
  • Use the SCHED_SPORADIC scheduling policy
  • When server has budget, runs at sched_priority, otherwise runs as a background server at sched_ss_low_priority
    • Set sched_ss_low_priority to be lower priority than real-time tasks, but possibly higher than other non-real-time tasks in the system
  • Also defines the replenishment period and the initial budget after replenishment
  • As usual with POSIX, applicable to fixed-priority systems only
Summary

• Use of sporadic server for scheduling aperiodic tasks – complex to implement, easy to prove correctness

• Scheduling sporadic tasks
  • In EDF systems – density test for correctness
  • In RM systems using sporadic server – complex rules for correctness, but intuition of behaviour straight-forward