Communications Theory

Networked Systems 3
Lecture 3

Lecture Outline

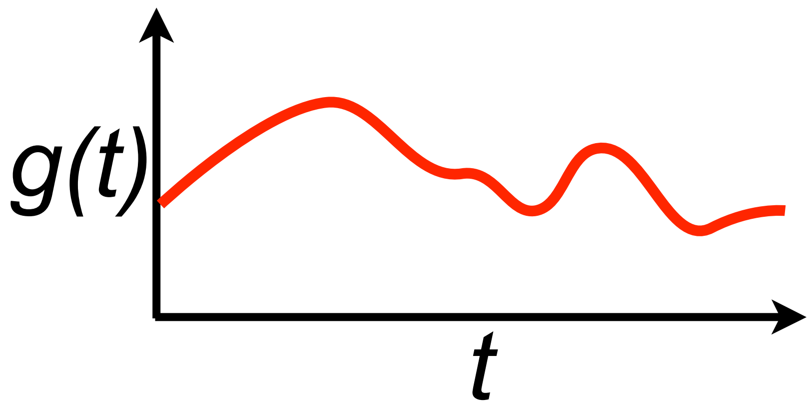
- Information content of signals
  - Time and frequency domain views of a signal
  - Fourier transform
  - Signal bandwidth as a proxy for information content
- Capacity of a channel
  - Perfect, noise free, channel
  - Channel subject to Gaussian noise
- Physical limits of communication

Information Theory

- Recall: communication happens when a *signal* is conveyed between source and destination via a channel
  - The channel has limited capacity
  - The amount of *information* in the signal determines if it will fit the channel
  - How to determine the amount of information in a signal, and the capacity of a channel?

How are Signals Conveyed?

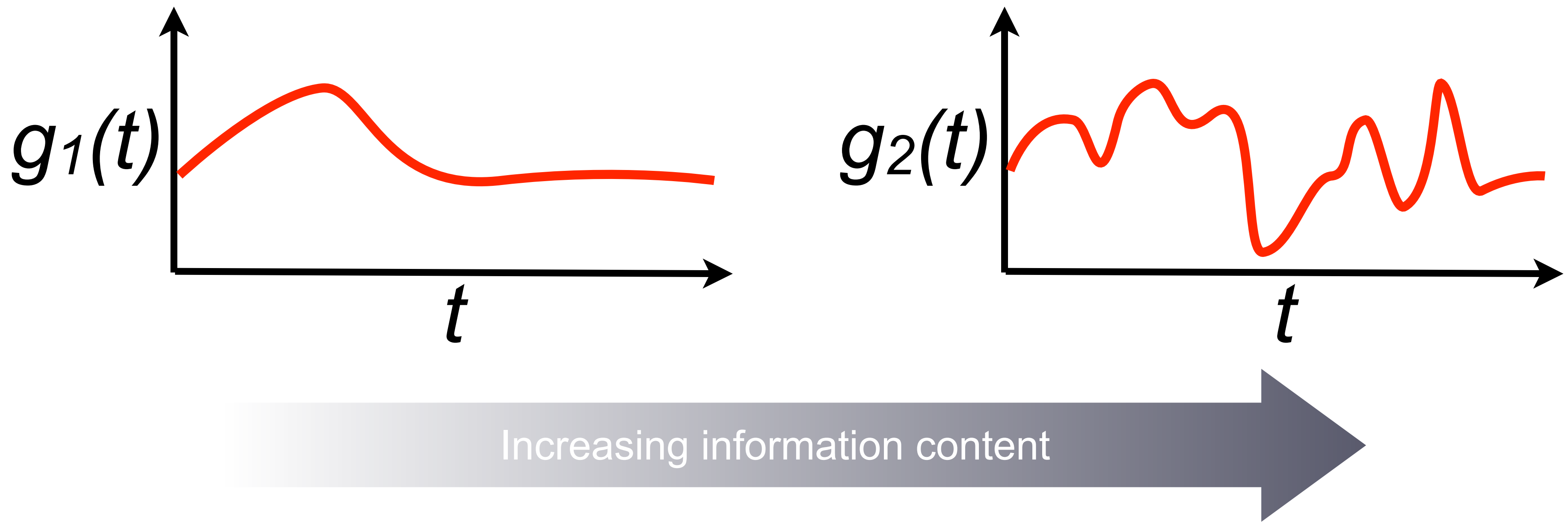
- Sender varies a physical property of the channel over time; receiver measures that property:
  - Voltage or current in an electrical cable
  - Modulation of a radio carrier
- Model as a mathematical function, *g(t)*



Time domain view

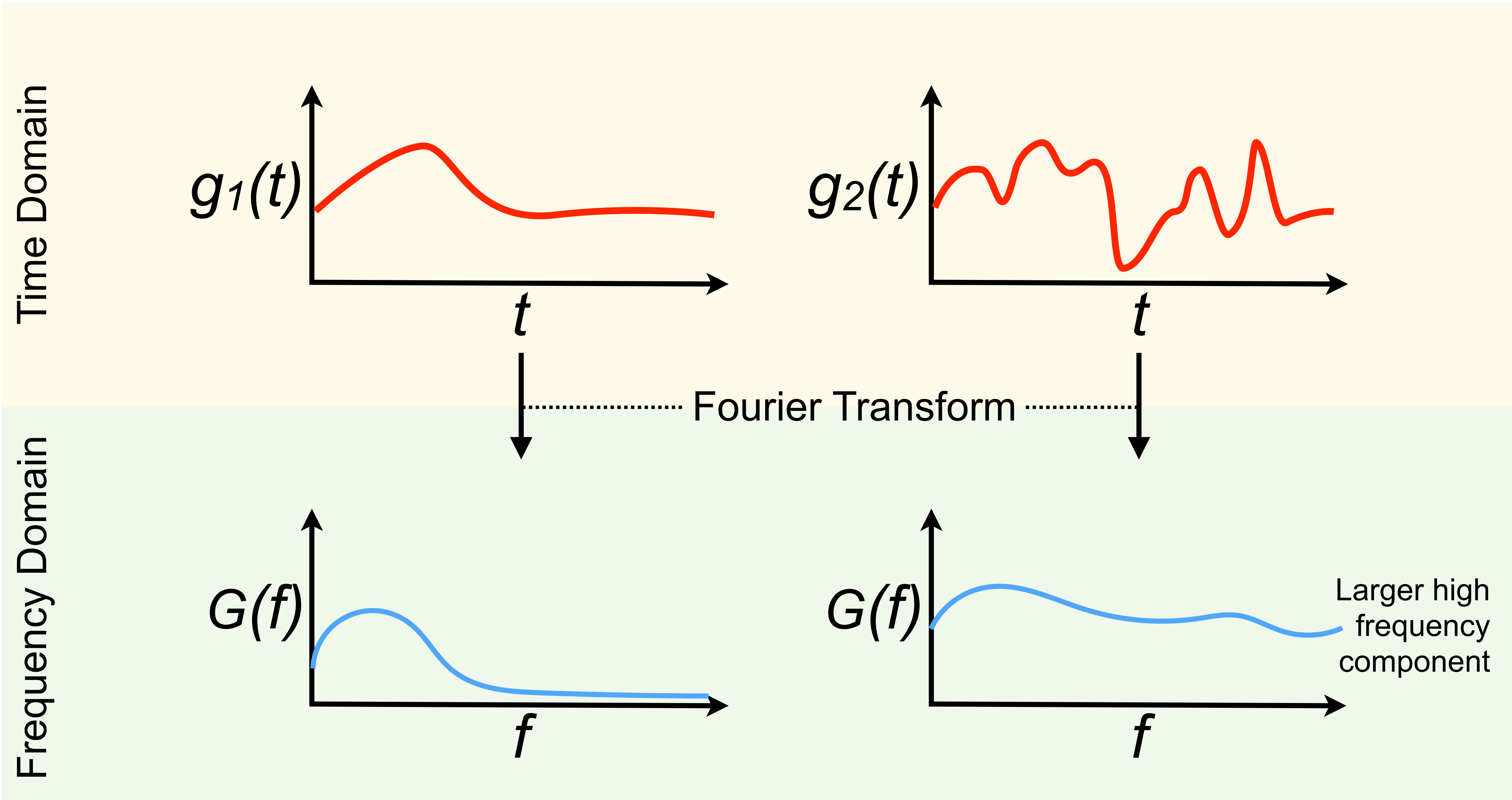
Information Content of a Signal

- More complex signals can convey more information



- Signal bandwidth acts as a proxy for information content
- More correctly: more complex signals convey more data; whether that data conveys more information depends on the efficiency of the encoding
- There are various ways of encoding information for transmission, with more or less efficiency

Time and Frequency Domains



More complex signals have a larger high frequency component: greater bandwidth

Fourier Analysis

- Mathematical method to derive frequency domain representation of a signal
  - Any well behaved periodic function can be constructed by summing a series of sines and cosines waves of varying frequency and amplitude

$$g(t) = \frac{1}{2}c + \sum_{n=1}^{\infty} a_n sin(2\pi n f t) + \sum_{n=1}^{\infty} b_n cos(2\pi n f t)$$

Amplitude

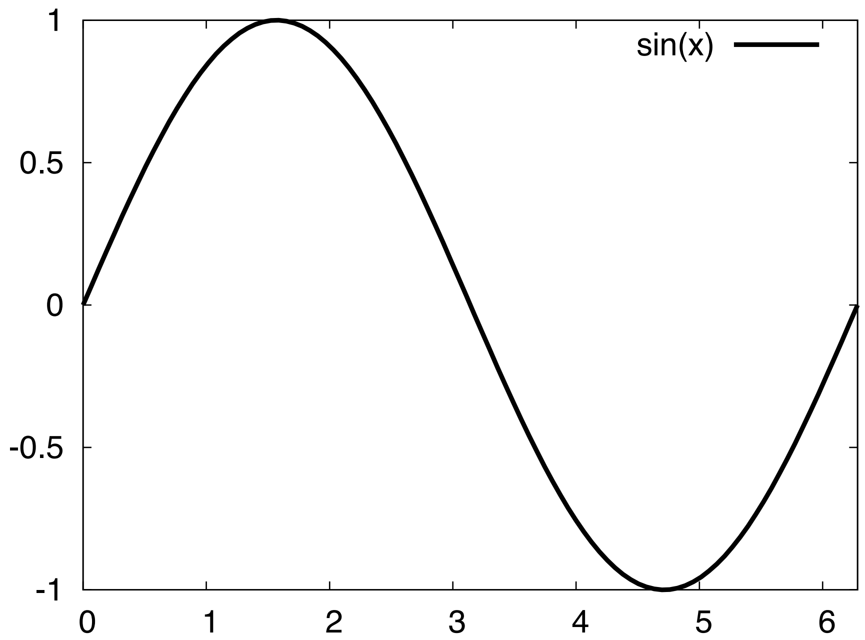
Frequency

  - Difference between the highest and lowest frequencies is the *signal bandwidth*

Source: Public domain

Joseph Fourier

Addition of Sine Waves



- More harmonics → more accuracy
    - Example is building a simple square wave, but any well-behaved function can be modelled

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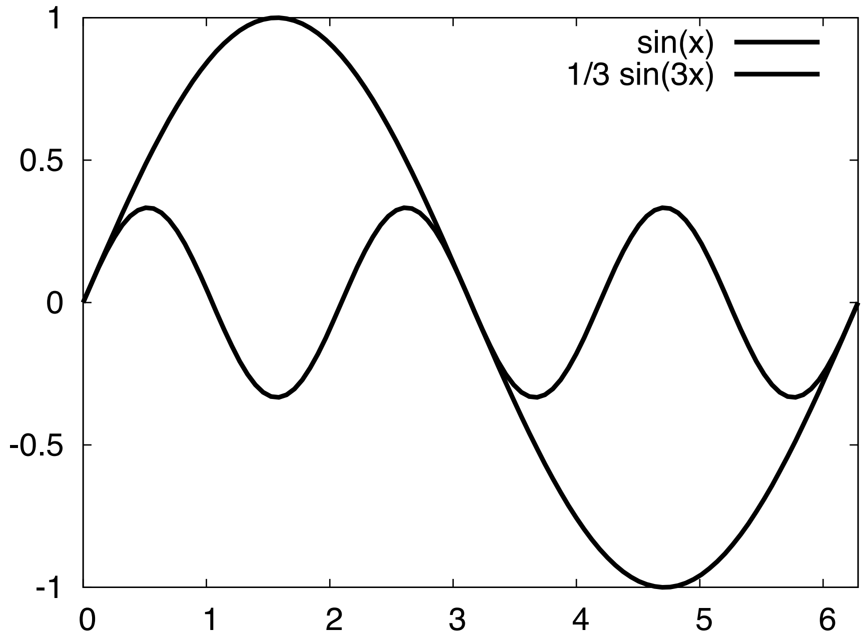

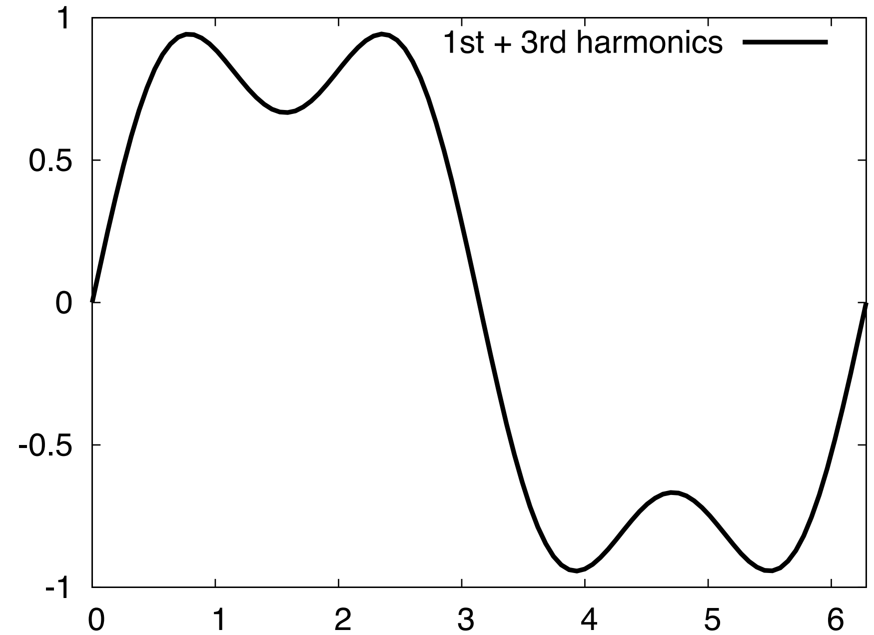

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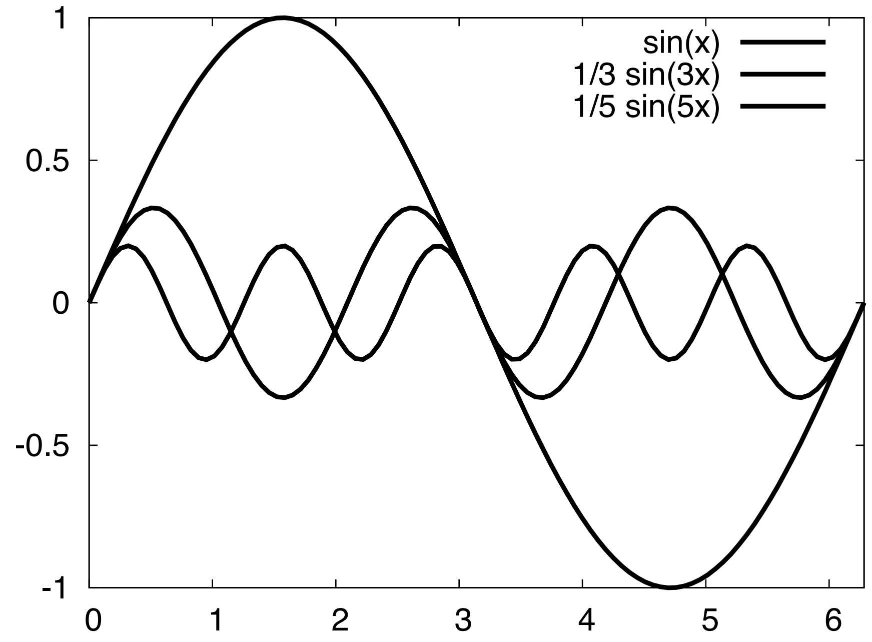

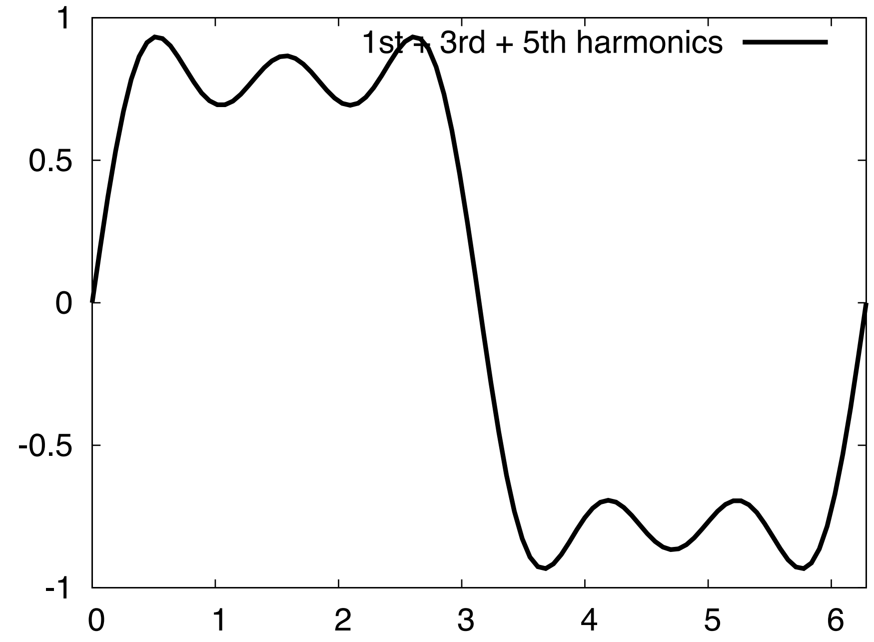

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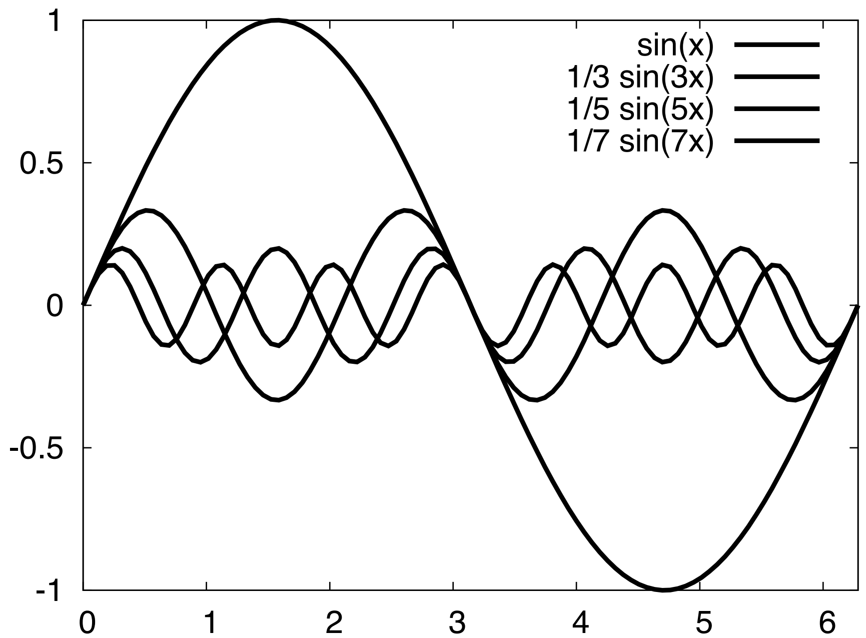

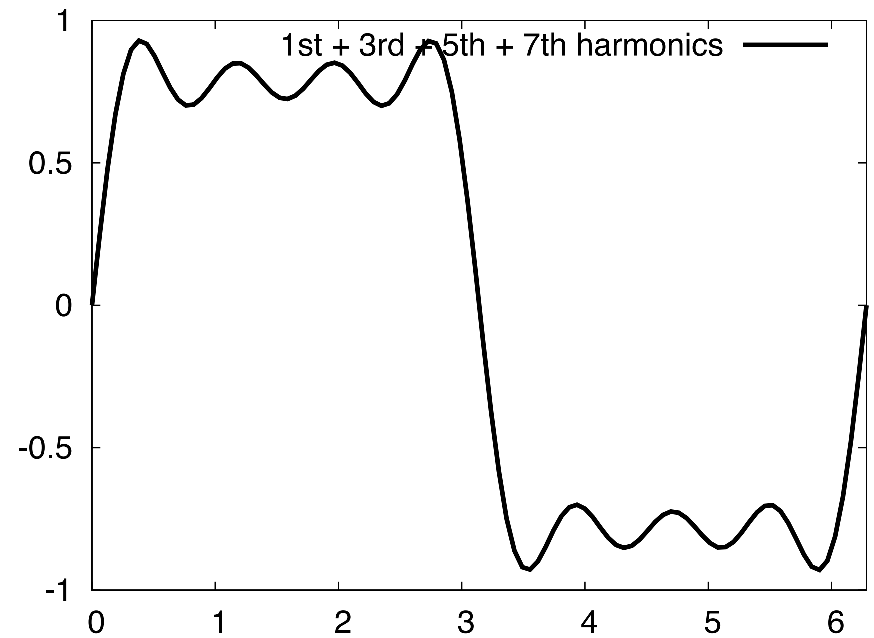

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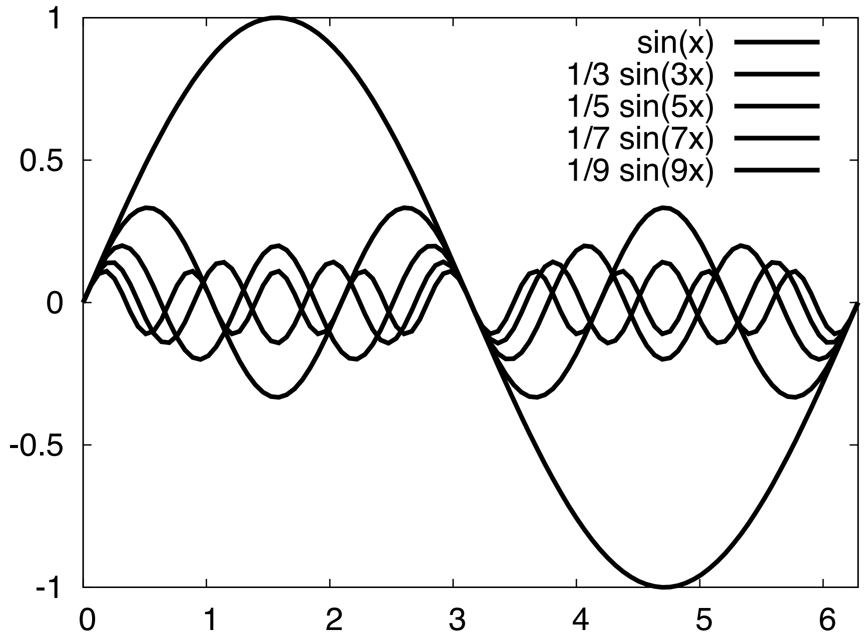

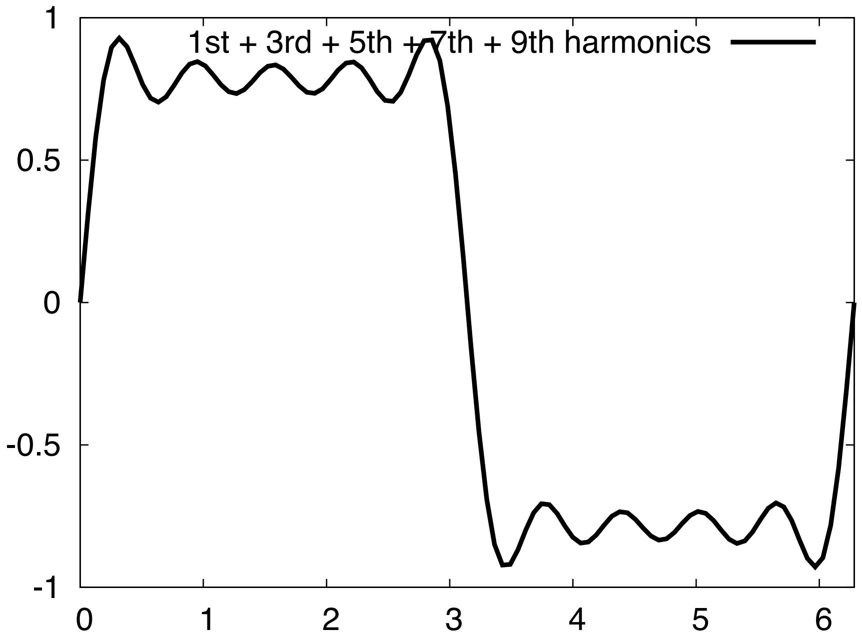

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Information Content

- Frequency domain view lets us visualise information content of a signal
  - More information → high frequency components
  - Limiting frequency range distorts signal – *alternatively* – signal content defines needed frequency range

Channel Bandwidth Limits

- Real channels cannot pass arbitrary frequencies
  - Fundamental limitations based on physical properties of the channel, design of the end points, etc.
  - The *channel bandwidth, H,* measures the frequency range (Hz) it can transport

- Implication: a channel can only convey a limited amount of information per unit time

Capacity of a Perfect Channel

- A channel's bandwidth determines the frequency range it can transport
- What about digital signals?
  - Sampling theorem: to accurately digitise an analogue signal, need *2H* samples per second
  - Maximum transmission rate of a digital signal depends on channel bandwidth: $R_{max} = 2H \log_2 V$
    - R_{max} = maximum transmission rate of channel (bits per second)
    - H = bandwidth
    - V = number of discrete values per symbol
  - Assumption: noise-free channel

Harry Nyquist, 1889-1976

Noise

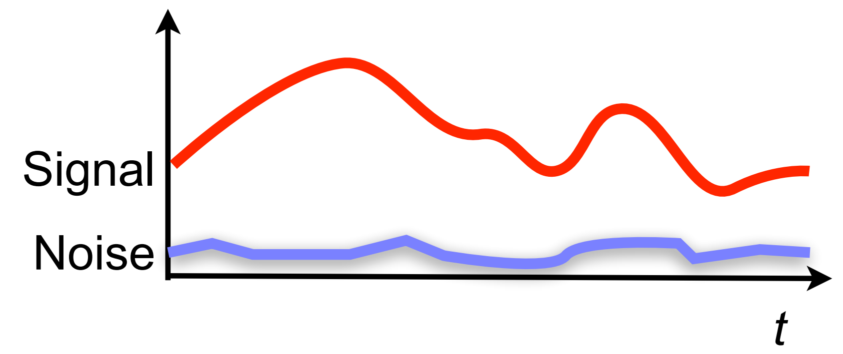
- Real world channels are subject to *noise*
- Many causes of noise:
  - Electrical interference
  - Cosmic radiation
  - Thermal noise

Different noise spectra

- Corrupts the signal: additive interference

Signal to Noise Ratio

- Can measure signal power, S, and noise floor, N, in a channel



- Gives signal-to-noise ratio: S/N
  - Typically quoted in decibels (dB), not directly
  - Signal-to-noise ratio in dB = $10 \log_{10} S/N$

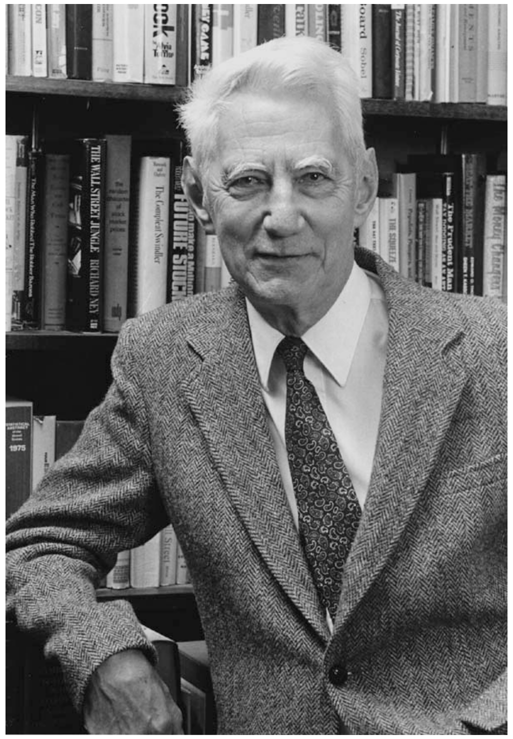
S/N	dB
2	3
10	10
100	20
1000	30

  - Example: ADSL modems report S/N ~30 for good quality phone lines: signal power 1000x greater than noise

Capacity of a Noisy Channel

- Capacity of noisy channel depends on type of noise
  - Uniform or bursty; affecting all or some frequencies

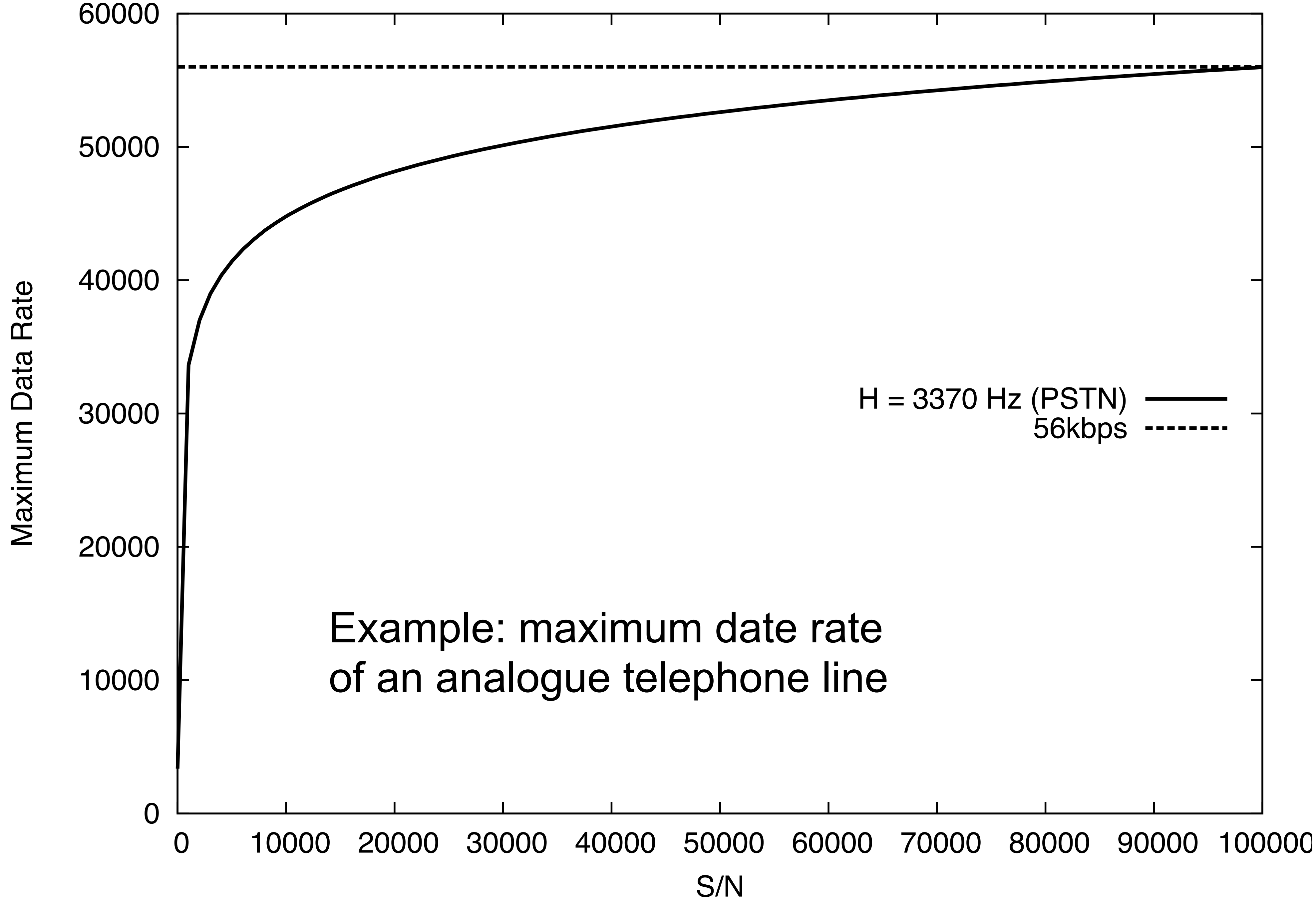
  - Simplest to model is *Gaussian noise:* uniform noise that impacts all frequencies equally
  - Maximum transmission rate of a channel subject to Gaussian noise: $R_{max} = H \log_2(1 + S/N)$



Source: AT&TT Bell Labs

Claude Shannon, 1916-2001

Capacity of a Noisy Channel

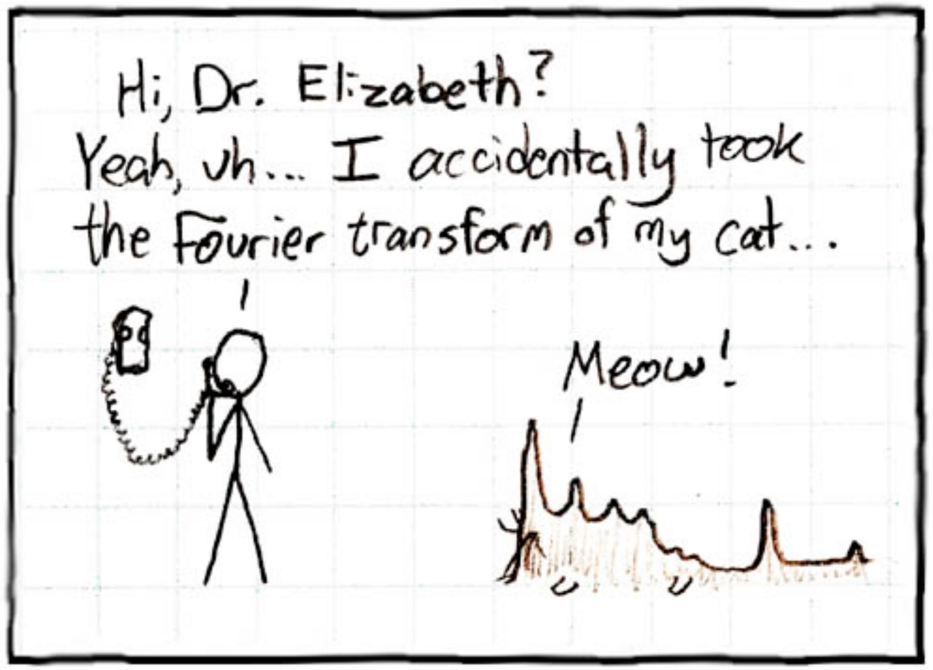


Implications

- Physical characteristics of channel limit amount of information that can be transferred
  - Bandwidth
  - Signal to noise ratio
- These are fundamental limits: might be reached with careful engineering, *but cannot be exceeded*

Summary

- More complex signals require more bandwidth
- Channels have limited bandwidth
- Physical limits on channel capacity due to noise imply physical limits on communication speed



Source: http://xkcd.com/26/