Communications Theory

Networked Systems 3
Lecture 4

Lecture Outline

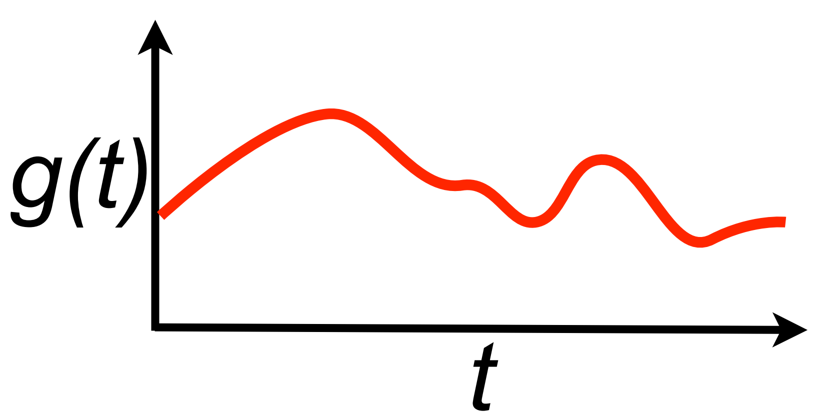
- Information content of signals
- Capacity of a channel
- Physical limits of communication

Information Theory

- Recall: communication happens when a *signal* is conveyed between source and destination via a channel
  - The channel has limited capacity
  - The amount of *information* in the signal determines if it will fit the channel
  - How to determine the amount of information in a signal, and the capacity of a channel?

How are Signals Conveyed?

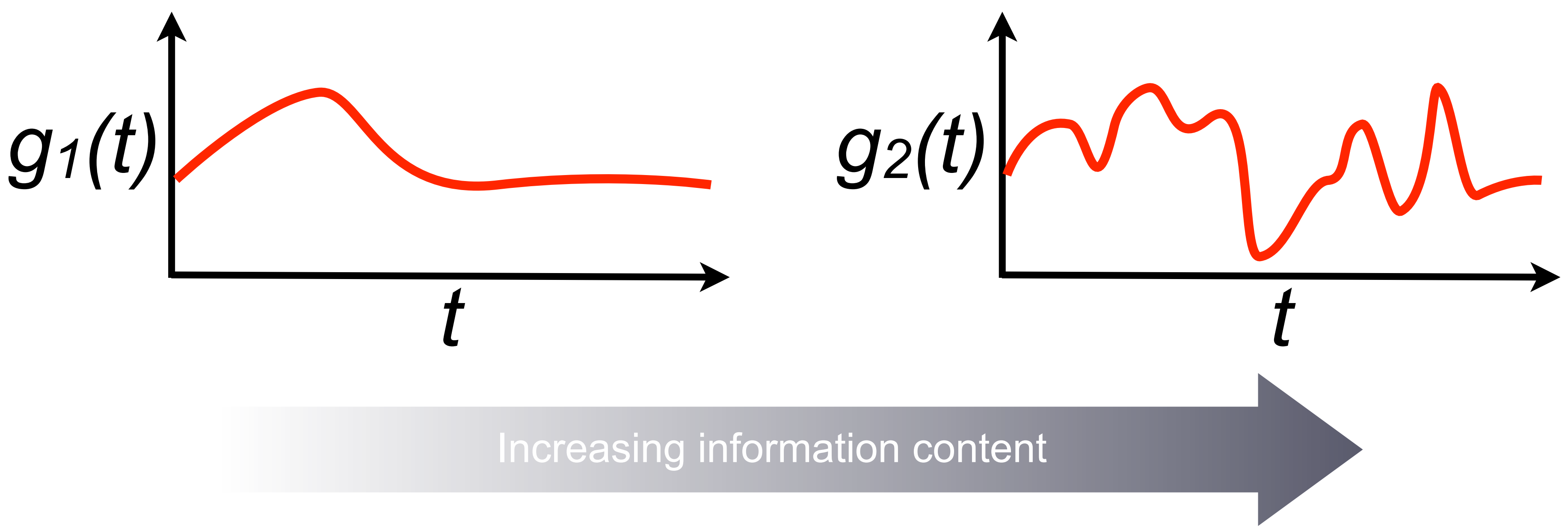
- Sender varies a physical property of the channel over time; receiver measures that property:
  - Voltage or current in an electrical cable
  - Modulation of a radio carrier
- Model as a mathematical function, *g(t)*



Time domain view

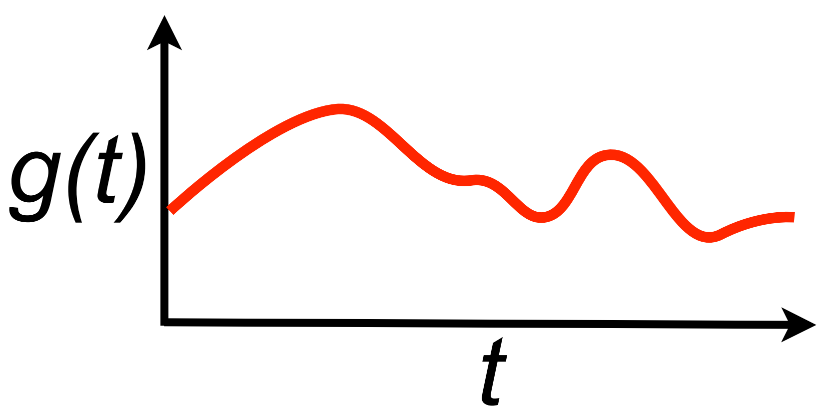
Information Content

- Intuition: a more complex signal carries more information



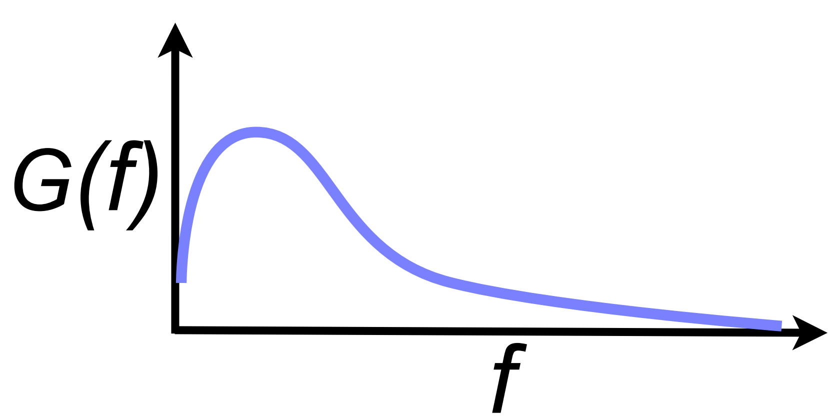
- Use *frequency domain analysis* to demonstrate this mathematically

Time and Frequency Domains



Time domain view

Fourier transform

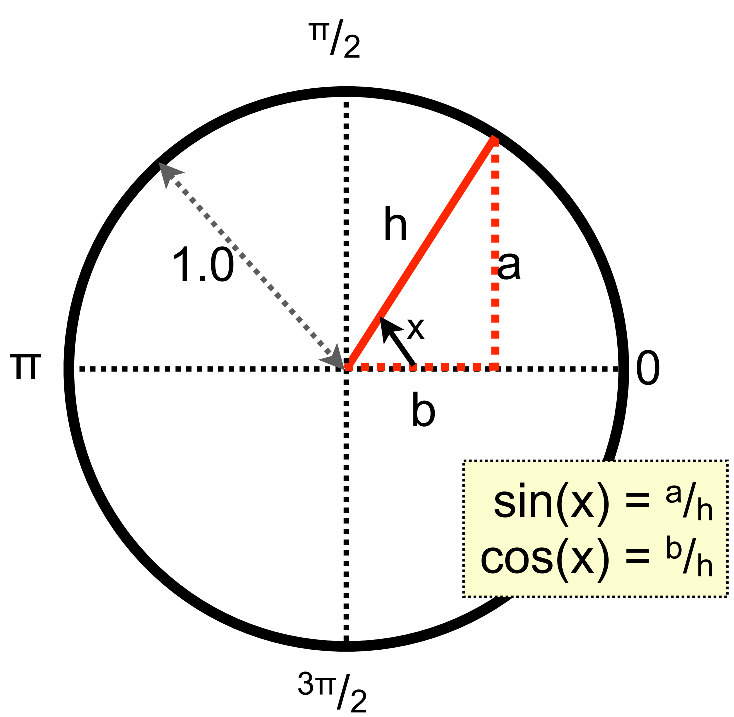


Frequency domain view

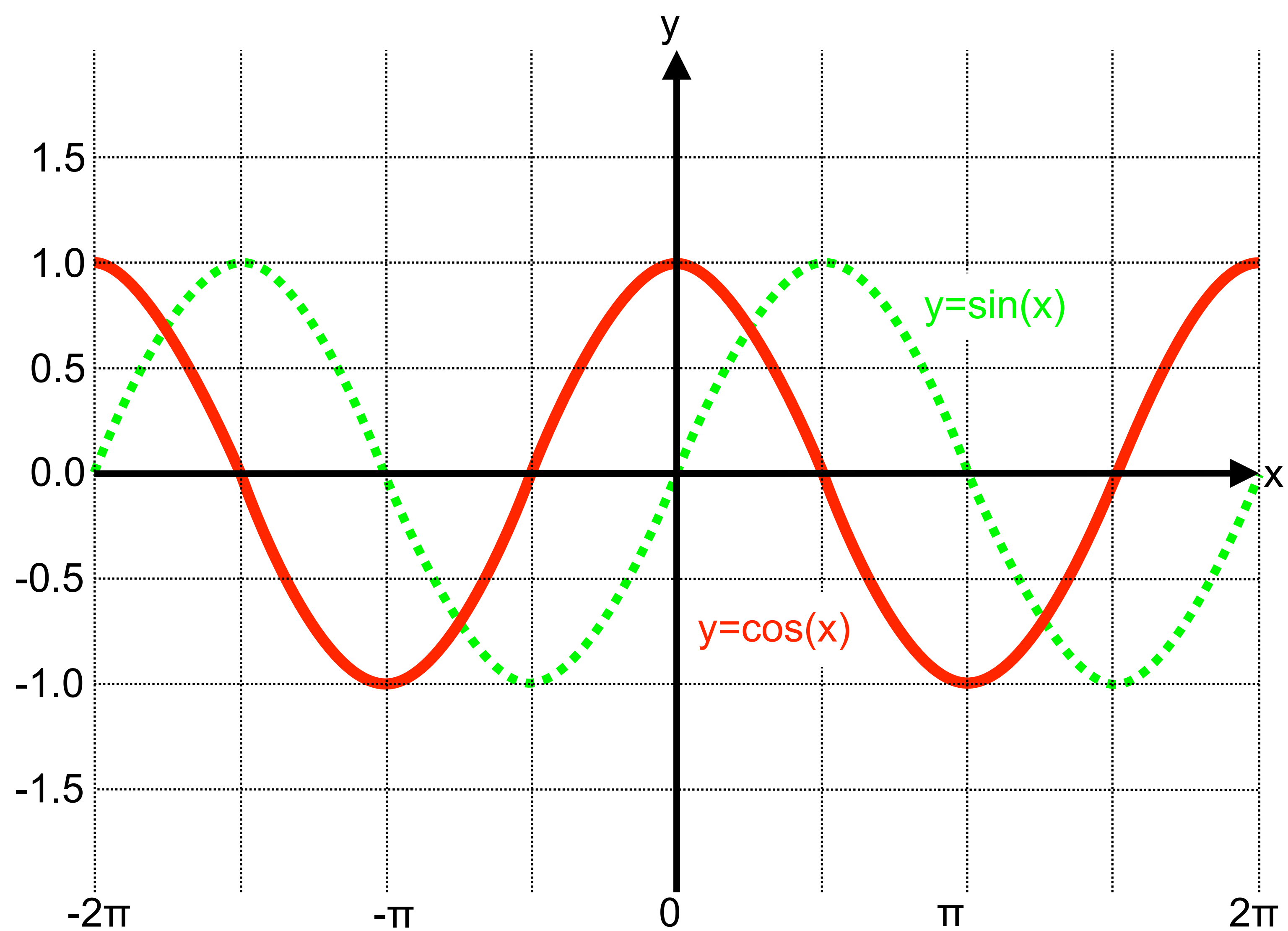
Sum of sines and cosines of varying frequency and amplitude

Complex signal → high frequency components [compare to music]

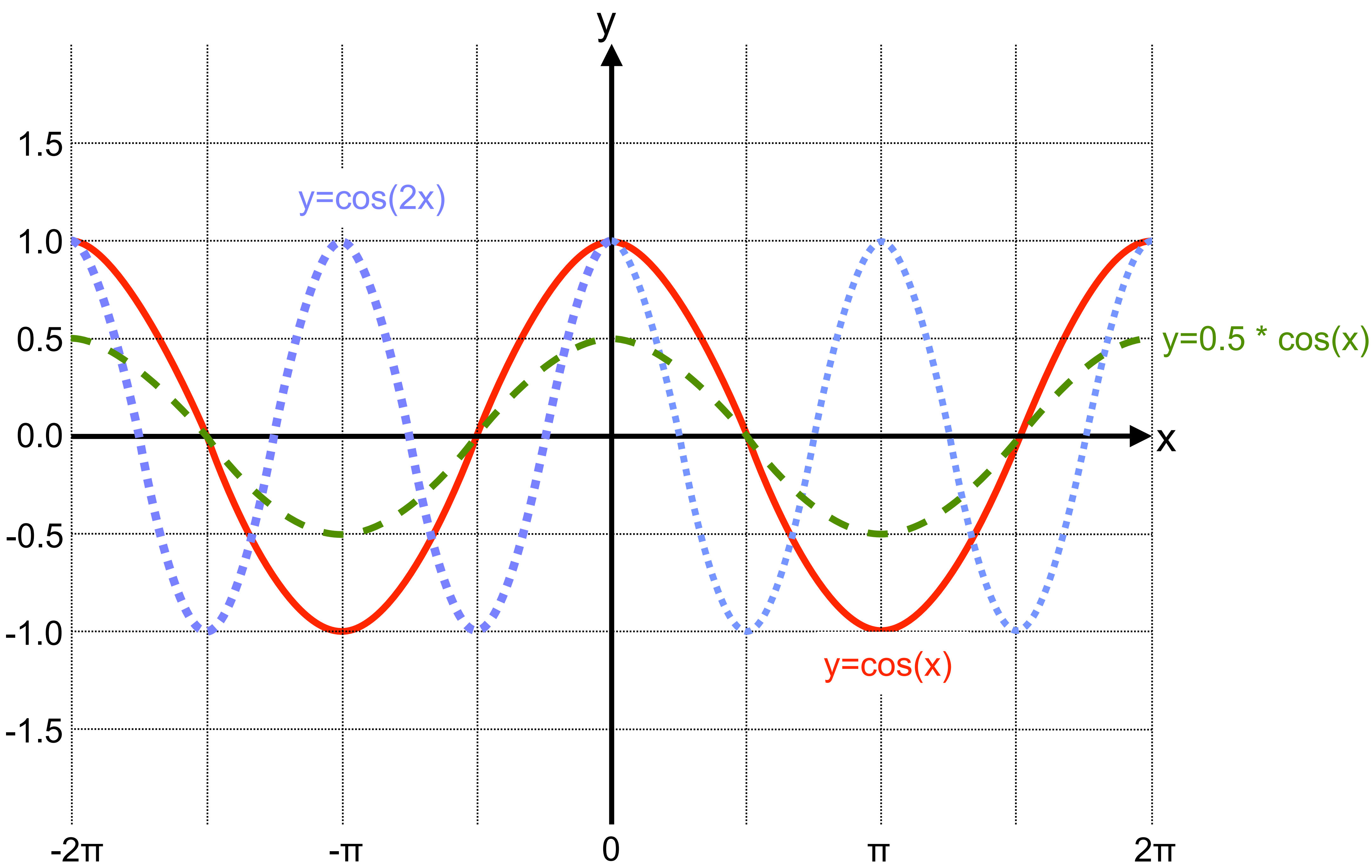
Sines and Cosines



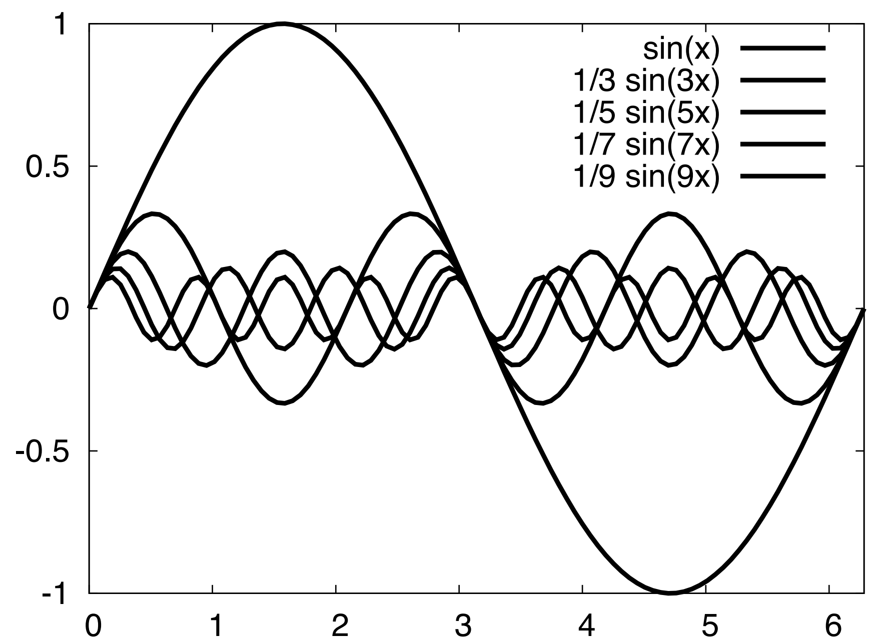
Sine wave: basis of frequency domain analysis

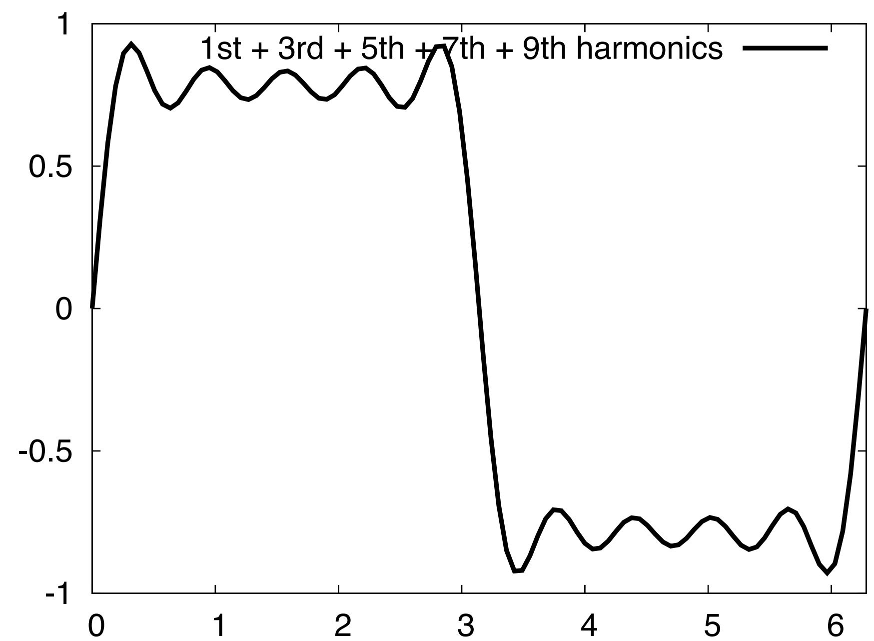


Frequency and Amplitude



Addition of Sine Waves





Can build more complex waveforms by adding together a sequence of sine waves

Infinite sequence: more harmonics → more accuracy

Fourier Analysis

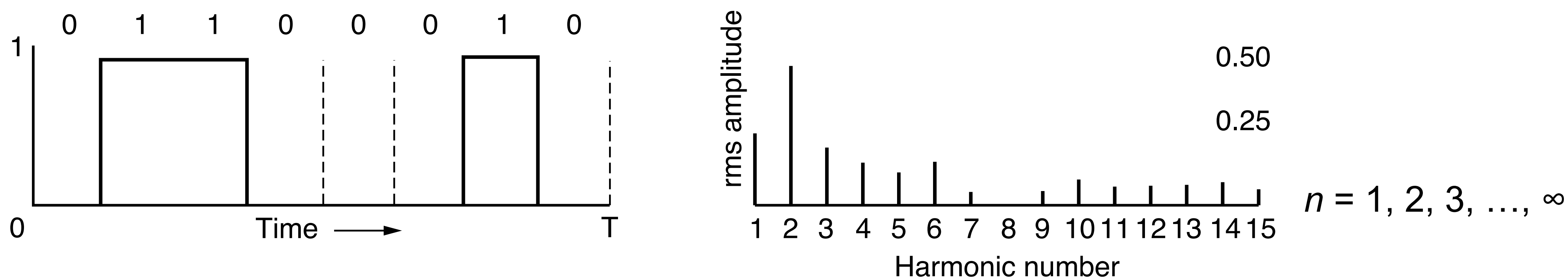
- Any well behaved periodic function can be constructed by summing a (possibly infinite) number of sines and cosines of varying frequency and amplitude
  - The *frequency domain* representation

Source: Public domain

Jean Baptiste Joseph Fourier,

$$g(t) = \frac{1}{2}c + \sum_{n=1}^{\infty} \underbrace{a_n}_{\text{Amplitude}} sin(2\pi \underbrace{n f}_{\text{Frequency}} t) + \sum_{n=1}^{\infty} b_n cos(2\pi n f t)$$

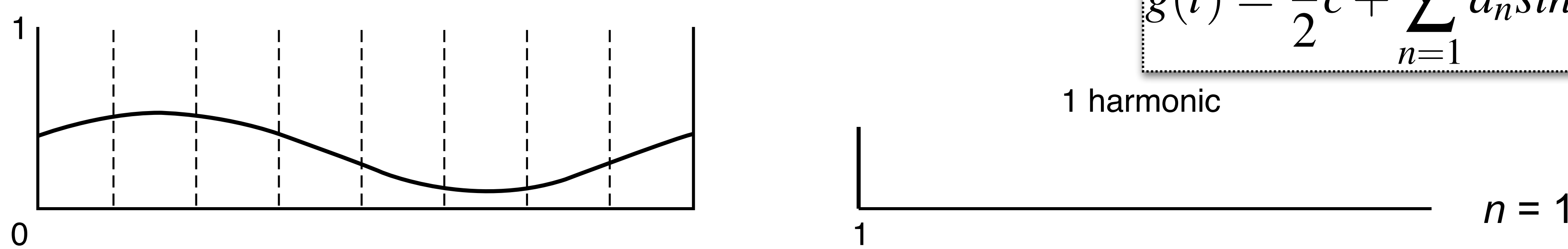
Fourier Analysis: Example (1)



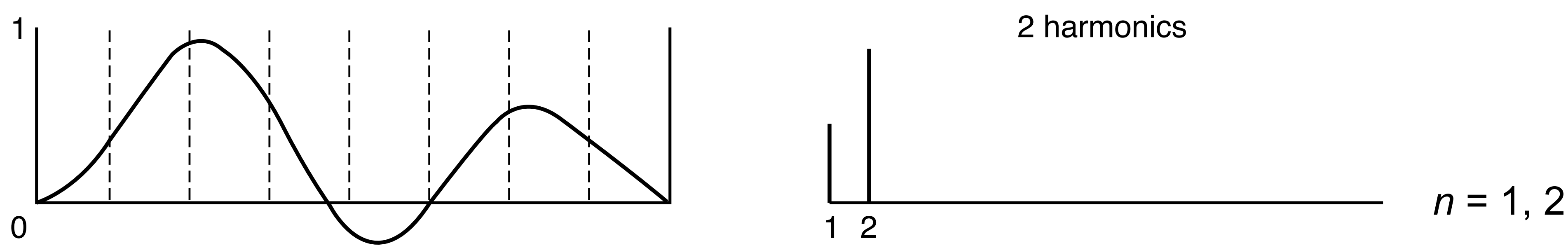
(a)

ASCII character "b"

$$g(t) = \frac{1}{2}c + \sum_{n=1}^{\infty} a_n sin(2\pi n f t) + \sum_{n=1}^{\infty} b_n cos(2\pi n f t)$$

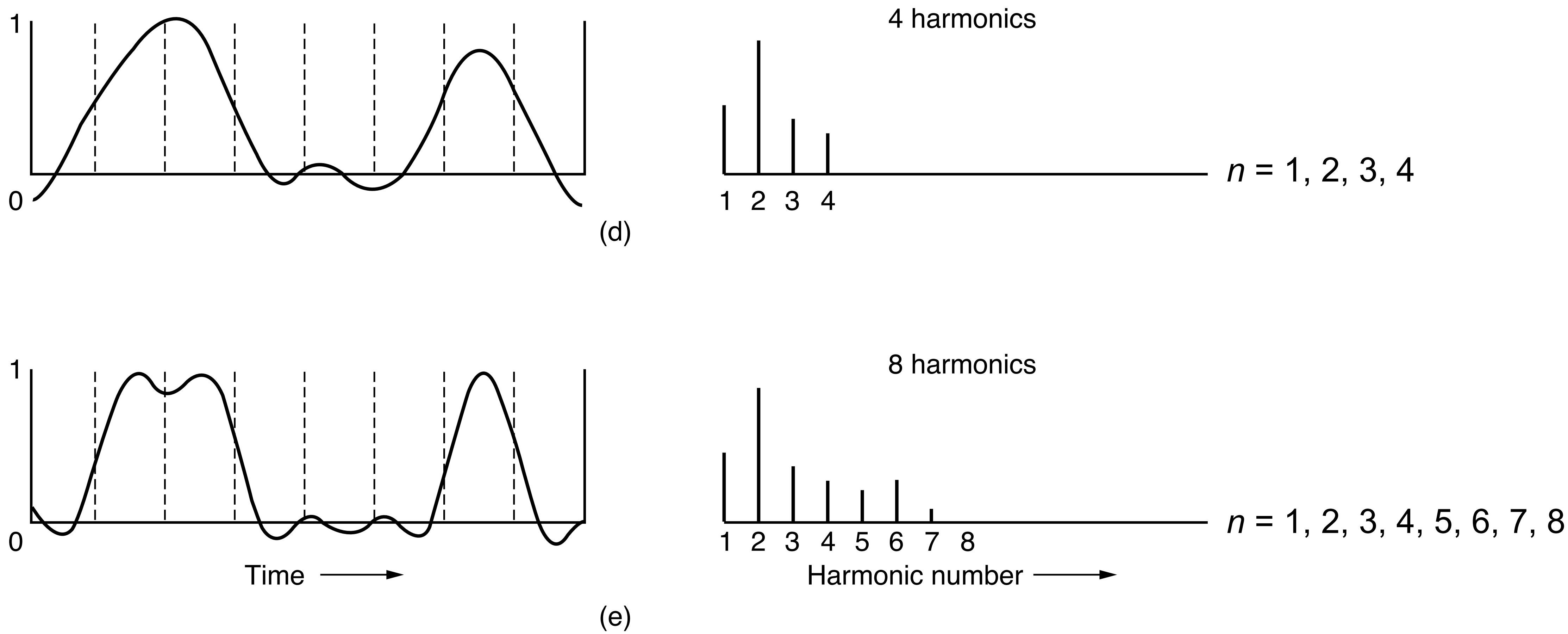


(b)



(c)

Fourier Analysis: Example (2)



Including more high frequency components (high harmonics) gives a more accurate representation

Information Content

- Frequency domain view allows us to visualise information content of a signal
  - More information → high frequency components
  - Limiting frequency range distorts signal – *alternatively* – signal content defines needed frequency range

Channel Bandwidth Limits

- Real channels cannot pass arbitrary frequencies
  - Fundamental limitations based on physical properties of the channel, design of the end points, etc.
  - The channel *bandwidth, H,* measures the frequency range (Hz) it can transport

- Implication: a channel can only convey a limited amount of information per unit time

Capacity of a Perfect Channel

- Bandwidth tells highest frequency that can be passed: *analogue signal*
- What about digital signals?
  - $R_{max} = 2H \log_2 V$
    - R_{max} = maximum data rate (bits per second)
    - H = bandwidth
    - V = number of discrete values per symbol
  - Assumption: noise-free channel

Source: IEEE

Harry Nyquist, 1889-1976

n	$\log_2(n)$
1	0.00
2	1.00
4	2.00
8	3.00

Noise

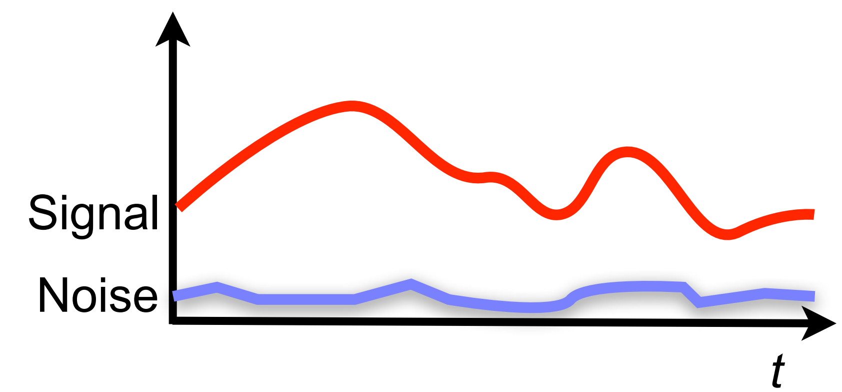
- Real world channels are subject to *noise*
- Many causes of noise:
  - Electrical interference
  - Cosmic radiation
  - Thermal noise

Different noise spectra

- Corrupts the signal: additive interference

Signal to Noise Ratio

- Can measure signal power, *S*, and noise power, *N*



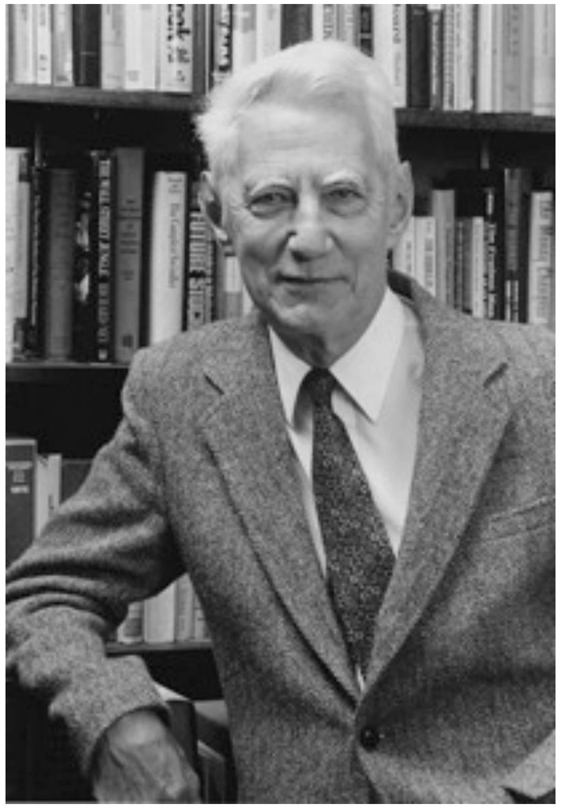
- Gives signal-to-noise ratio: *S/N*
  - Typically quoted in decibels (dB), not directly
  - Signal-to-noise ratio in dB = $10 \log_{10} S/N$

S/N	dB
2	3
10	10
100	20
1000	30

Capacity of a Noisy Channel

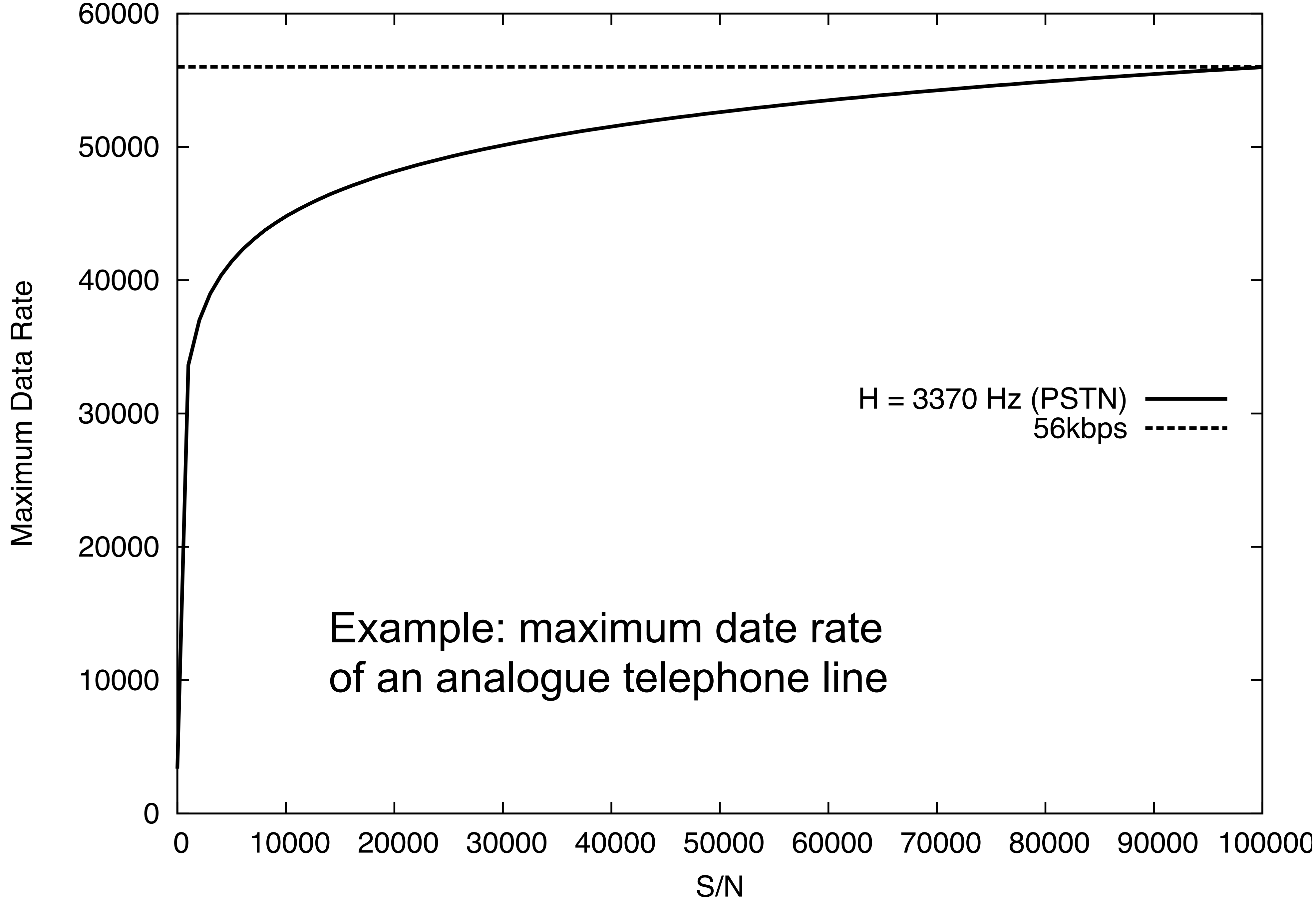
- $R_{max} = H \log_2(1 + S/N)$
  - R_{max} = maximum data rate (bits per second)
  - H = bandwidth

- Note:
  - Channel subject to white noise
  - Irrespective of number of discrete values per symbol



Claude Shannon, 1916-2001

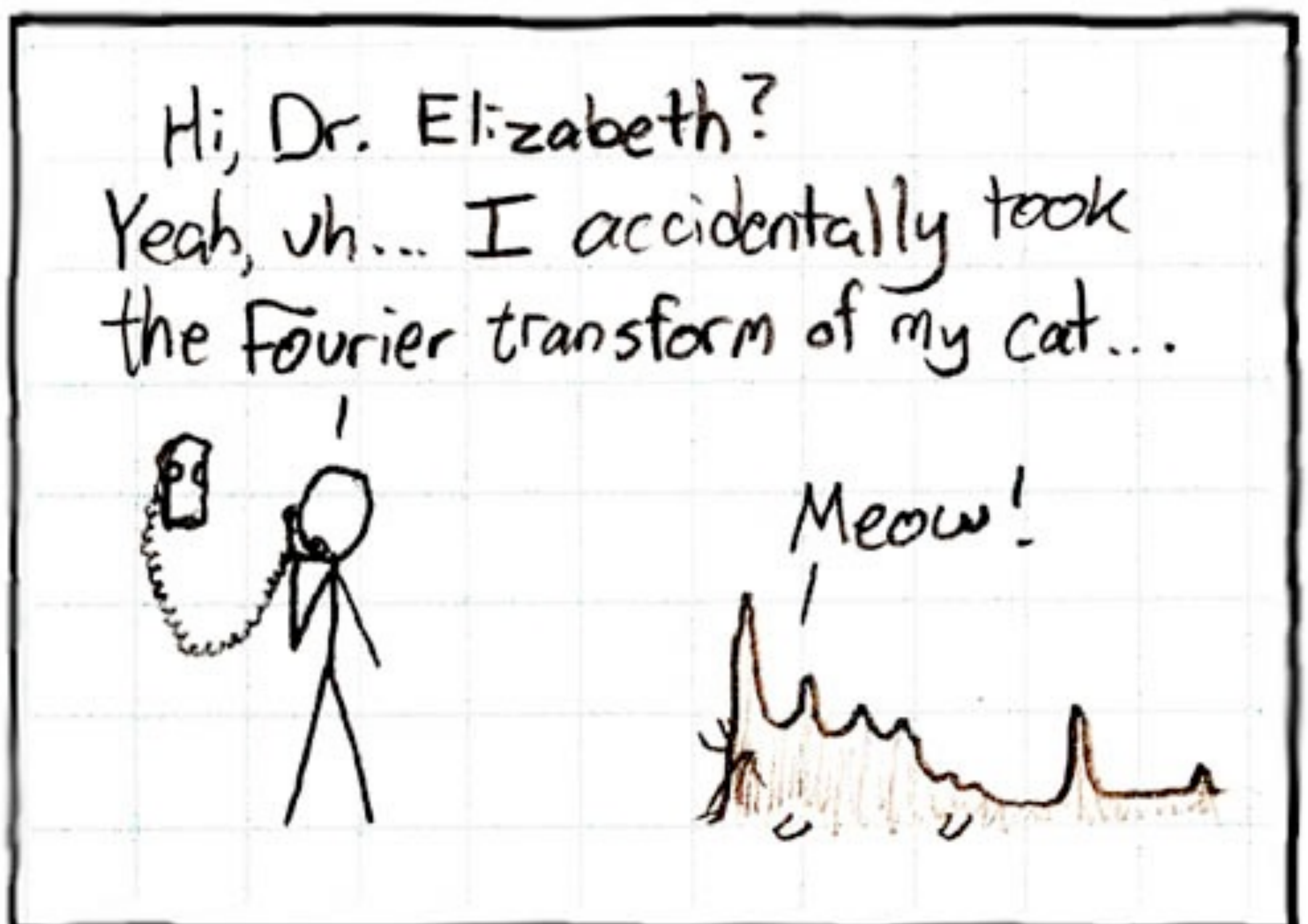
Capacity of a Noisy Channel



Implications

- Physical characteristics of channel limit amount of information that can be transferred
    - Bandwidth
    - Signal to noise ratio
- These are fundamental limits: might be reached with careful engineering, *but cannot be exceeded*

Questions?



Source: http://xkcd.com/26/