Priority-driven Scheduling of Periodic Tasks (2)

Real-Time and Embedded Systems (M)
Lecture 6
Lecture Outline

• Schedulability tests for fixed-priority systems
  – Conditions for optimality and schedulability
  – General schedulability tests and time demand analysis

• Practical factors
  – Non-preemptable regions
  – Self-suspension
  – Context switches
  – Limited priority levels

[Continues from material in lecture 5, with the same assumptions]
Optimality and Schedulability

- You will recall:
  - EDF and LST dynamic priority scheduling optimal:
    - Always produce a feasible schedule if one exists – on a single processor, as long as preemption is allowed and jobs do not contend for resources
    - Lecture 3 + confirmation last lecture: $U_{EDF} = 1$
  
- Fixed priority algorithms non-optimal in general:
  - e.g. RM and DM sometimes fail to schedule tasks that can be scheduled using other algorithms
  - Proof: Misses deadlines unless relative priority changes; cannot be scheduled using RM
  
- Hence introduced schedulability tests in lecture 5
Optimality of RM and DM Algorithms

• Fixed priority algorithms can be optimal in restricted systems

• Example:
  – RM and DM are optimal in simply periodic systems
  – A system of periodic tasks is simply periodic if the period of each task is an integer multiple of the period of the other tasks:
    \[ p_k = n \cdot p_i \]
    where \( p_i < p_k \) and \( n \) is a positive integer; for all \( T_i \) and \( T_k \)
  – True for many real-world systems, e.g. the helicopter flight control system discussed in lecture 1
Optimality of RM and DM Algorithms

• Theorem: A system of simply periodic, independent, preemptable tasks with $D_i \geq p_i$ is schedulable on one processor using the RM algorithm if and only if $U \leq 1$
  – Corollary: The same is true for the DM algorithm

• Proof:
  – A simply periodic system, assume tasks in phase
    • Worst case execution time occurs when tasks in phase
  – $T_i$ misses deadline at time $t$ where $t$ is an integer multiple of $p_i$
    • Again, worst case $\Rightarrow D_i = p_i$
  – Simply periodic $\Rightarrow t$ integer multiple of periods of all higher priority tasks
  – Total time required to complete jobs with deadline $\leq t$ is $\sum_{k=1}^{i} \frac{e_k}{p_k} t = t \cdot U_i$
  – Only fails when $U_i > 1$
Schedulability of Fixed-Priority Tasks

• Identified several simple schedulability tests for fixed-priority scheduling:
  – A system of $n$ independent preemptable periodic tasks with $D_i = p_i$ can be feasibly scheduled on one processor using RM iff $U \leq n \cdot (2^{1/n} - 1)$
  – A system of *simply periodic* independent preemptable tasks with $D_i \geq p_i$ is schedulable on one processor using the RM algorithm iff $U \leq 1$
  – [similar results for DM]

• But: there are algorithms and regions of operation where we don’t have a schedulability test and must resort to exhaustive simulation
  – Is there a more general schedulability test?
  – Yes, extend the approach taken for simply periodic system schedulability
**Fixed-Priority Tasks: Schedulability Test**

- Fixed priority algorithms are predictable and do not suffer from *scheduling anomalies*
  - The worst case execution time of the system occurs with the worst case execution time of the jobs, unlike dynamic priority algorithms which can exhibit anomalous behaviour

  [See also lecture 3]

- Use this as the basis for a general schedulability test:
  - Find the *critical instant* when the system is most loaded, and has its worst response time
  - Use *time demand analysis* to determine if the system is schedulable at that instant
  - Prove that, if a fixed-priority system is schedulable at the critical instant, it is always schedulable
Finding the Critical Instant

• A critical instant for a job is the worst-case release time for that job, taking into account all jobs that have higher priority
  – i.e. a job released at the same instant as all jobs with higher priority are released, and must wait for all those jobs to complete before it executes
  – The response time of a job in $T_i$ released at a critical instant is called the maximum (possible) response time, and is denoted by $W_i$

• The schedulability test involves checking each task in turn, to verify that it can be scheduled when started at a critical instant
  – If schedulable at all critical instants, will work at other times
  – More work than the test for maximum schedulable utilization, but less than an exhaustive simulation
Finding the Critical Instant

• A critical instant of a task $T_i$ is a time instant such that:
  
  If $w_{i,k} \leq D_{i,k}$ for every $J_{i,k}$ in $T_i$ then
  
  The job released at that instant has the maximum response time of all jobs in $T_i$
  
  and $W_i = w_{i,k}$
  
  else if $\exists J_{i,k} : w_{i,k} > D_{i,k}$ then
  
  The job released at that instant has response time $> D$
  
  where $w_{i,k}$ is the response time of the job

• Theorem: In a fixed-priority system where every job completes before the next job in the same task is released, a critical instant occurs when one of its jobs $J_{i,c}$ is released at the same time with a job from every higher-priority task.
  
  – Intuitively obvious, but proved in the book

All jobs meet deadlines, but this instant is when the job with the slowest response is started

If some jobs don’t meet deadlines, this is one of those jobs
Finding the Critical Instant: Example

3 tasks scheduled using rate-monotonic

- Response times of jobs in $T_2$ are:
  
  $r_{2,1} = 0.8, r_{2,3} = 0.3, r_{2,3} = 0.2, r_{2,4} = 0.3, r_{2,5} = 0.8, \ldots$

Therefore critical instants of $T_2$ are $t = 0$ and $t = 10$
Using the Critical Instant

- Having determined the critical instants, show that for each job $J_{i,c}$ released at a critical instant, that job and all higher priority tasks complete executing before their relative deadlines.
- If so, the entire system be schedulable…

- That is: don’t simulate the entire system, simply show that it has correct characteristics following a critical instant.
  - This process is called *time demand analysis*.
Time-Demand Analysis

• Compute the total demand for processor time by a job released at a critical instant of a task, and by all the higher-priority tasks, as a function of time from the critical instant.

• Check if this demand can be met before the deadline of the job:
  – Consider one task, $T_i$, at a time, starting highest priority and working down to lowest priority.
  – Focus on a job, $J_i$, in $T_i$, where the release time, $t_0$, of that job is a critical instant of $T_i$.
  – At time $t_0 + t$ for $t \geq 0$, the processor time demand $w_i(t)$ for this job and all higher-priority jobs released in $[t_0, t]$ is:

$$w_i(t) = e_i + \sum_{k=1}^{i-1} \left[ \frac{t}{p_k} \right] e_k \quad \text{for} \quad 0 < t \leq p_i$$

$w_i(t)$ = the time-demand function

Execution time of job $J_i$

Execution time of higher priority jobs started during this interval
Time-Demand Analysis

• Compare the time demand, $w_i(t)$, with the available time, $t$:
  – If $w_i(t) \leq t$ for some $t \leq D_i$, the job, $J_i$, meets its deadline, $t_0 + D_i$
  – If $w_i(t) > t$ for all $0 < t \leq D_i$ then the task probably cannot complete by its deadline; and the system likely cannot be scheduled using a fixed priority algorithm
    • Note that this is a sufficient condition, but not a necessary condition. Simulation may show that the critical instant never occurs in practice, so the system could be feasible…

• Use this method to check that all tasks are schedulable if released at their critical instants; if so conclude the entire system can be scheduled
Rate Monotonic:  
\( T_1 = (3, 1), T_2 = (5, 2), T_3 = (10, 2) \)  
\( U = 0.933 \)

The time-demand functions \( w_1(t), w_2(t) \) and \( w_3(t) \) are not above \( t \) at their deadline \( \Rightarrow \) system can be scheduled

Exercise: simulate the system to check this!
Time-Demand Analysis

• The time-demand function $w_i(t)$ is a staircase function
  – Steps in the time-demand for a task occur at multiples of the period for higher-priority tasks
  – The value of $w_i(t) - t$ linearly decreases from a step until the next step

• If our interest is the schedulability of a task, it suffices to check if $w_i(t) \leq t$ at the time instants when a higher-priority job is released

• Our schedulability test becomes:
  – Compute $w_i(t)$
  – Check whether $w_i(t) \leq t$ is satisfied at any of the instants $t = j \cdot p_k$
    where $k = 1, 2, \ldots, i$
    $j = 1, 2, \ldots, \lfloor \min(p_i, D_i)/p_k \rfloor$
Time-Demand Analysis: Summary

• Time-demand analysis schedulability test is more complex than the schedulable utilization test, but more general
  – Works for any fixed-priority scheduling algorithm, provided the tasks have short response time (i.e. \( p_i < D_i \))
  – Can be extended to tasks with arbitrary deadlines (see book)
  – Only a sufficient test: guarantees that schedulable results are correct, but requires further testing to validate a result of not schedulable

• Alternative approach: simulate the behaviour of tasks released at the critical instants, up to the largest period of the tasks
  – Still involves simulation, but less complex than an exhaustive simulation of the system behaviour
  – Worst-case simulation method
Practical Factors

• We have assumed that:
  – Jobs are preemptable at any time
  – Jobs never suspend themselves
  – Each job has distinct priority
  – The scheduler is event driven and acts immediately

• These assumptions are often not valid… how does this affect the system?
Blocking and Priority Inversion

- A ready job is *blocked* when it is prevented from executing by a lower-priority job; a *priority inversion* is when a lower-priority job executes while a higher-priority job is blocked.

- These occur because some jobs cannot be pre-empted:
  - Many reasons why a job may have non-preemptable sections
    - Critical section over a resource
    - Some system calls are non-preemptable
    - Disk scheduling
  - If a job becomes non-preemptable, priority inversions may occur, these may cause a higher priority task to miss its deadline
  - When attempting to determine if a task meets all of its deadlines, must consider not only all the tasks that have higher priorities, but also non-preemptable regions of lower-priority tasks
    - Add the blocking time in when calculating if a task is schedulable.
Self-Suspension and Context Switches

• Self-suspension
  – A job may invoke an external operation (e.g. request an I/O operation), during which time it is suspended
  – This means the task is no longer strictly periodic… again need to take into account self-suspension time when calculating a schedule

• Context Switches
  – Assume maximum number of context switches $K_i$ for a job in $T_i$ is known; each takes $t_{CS}$ time units
  – Compensate by setting execution time of each job, $e_{\text{actual}} = e + 2t_{CS}$
    (more if jobs self-suspend, since additional context switches)
Tick Scheduling

• All of our previous discussion of priority-driven scheduling was driven by job release and job completion events
• Alternatively, can perform priority-driven scheduling at periodic events (timer interrupts) generated by a hardware clock
  – i.e. tick (or time-based) scheduling
• Additional factors to account for in schedulability analysis
  – The fact that a job is ready to execute will not be noticed and acted upon until the next clock interrupt; this will delay the completion of the job
  – A ready job that is yet to be noticed by the scheduler must be held somewhere other than the ready job queue, the pending job queue
  – When the scheduler executes, it moves jobs in the pending queue to the ready queue according to their priorities; once in ready queue, the jobs execute in priority order
Practical Factors

• Clear that non-ideal behaviour can affect the schedulability of a system
• Have touched on how – more details later in the module
Summary

• Have discussed fixed-priority scheduling of periodic tasks:
  – Optimality of RM and DM
  – More general schedulability tests and time-demand analysis
•.Outlined practical factors that affect real-world periodic systems

• Tutorial tomorrow will recap the material from lectures 5 and 6

• Problem set 2 now available: due at 1:00pm on 1st February